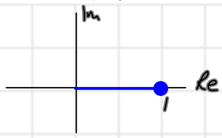
(i) Solve $z^3 = 1$, leave answers in cartesian form.

$$\Rightarrow z = 1^{\frac{1}{3}}$$



Polar Form

$$r = 1 \qquad \theta = 0$$

$$z = \left[1(\cos 0 + i \sin 0)\right]^{\frac{1}{3}}$$

$$= \left[\cos 0 + i \sin 0\right]^{\frac{1}{3}}$$

General Polar Form

$$= \left[\cos(0 + 2n\pi) + (i \sin 0 + 2n\pi)\right]^{\frac{1}{3}}$$

de Moivre

$$= \cos \tfrac{1}{3}(0 + 2n\pi) + i \sin \tfrac{1}{3}(0 + 2n\pi)$$

z^3 has 3 roots

$$n = 0 \Rightarrow z = \cos 0 + i \sin 0 = 1$$

$$n = 1 \Rightarrow z = \cos \tfrac{1}{3}(0 + 2\pi) + i \sin \tfrac{1}{3}(0 + 2\pi)$$
$$= \cos \tfrac{2\pi}{3} + i \sin \tfrac{2\pi}{3} = -\tfrac{1}{2} + \tfrac{\sqrt{3}}{2} i$$

$$n = 2 \Rightarrow z = \cos \tfrac{1}{3}(0 + 4\pi) + i \sin \tfrac{1}{3}(0 + 4\pi)$$
$$= \cos \tfrac{4\pi}{3} + i \sin \tfrac{4\pi}{3} = -\tfrac{1}{2} - \tfrac{\sqrt{3}}{2} i$$

(ii) If one of these solutions is ω show that another is $(\omega)^2$.

let $-\frac{1}{2} + \frac{\sqrt{3}}{2} i = \omega$

$(a+b)^2 = a^2 + 2ab + b^2$

$$\omega^2 = \frac{1}{4} - \frac{2\sqrt{3}}{4} i \mp \frac{3}{4} \cancel{i^2}$$

$$= \frac{-1}{2} - \frac{\sqrt{3}}{4}$$

yes this is the third solution in part (i).

(iii) what quadratic equation has roots ω and $\frac{1}{\omega}$?

$\omega = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$

notice difference of 2 squares

$$\frac{1}{\omega} = \frac{1}{(-\frac{1}{2}+\frac{\sqrt{3}}{2}i)} \cdot \frac{(-\frac{1}{2}-\frac{\sqrt{3}}{2}i)}{(-\frac{1}{2}-\frac{\sqrt{3}}{2}i)}$$

$$= \frac{-\frac{1}{2}-\frac{\sqrt{3}}{2}i}{\frac{1}{4} \pm \frac{3}{4}\cancel{i^2}} = \frac{-\frac{1}{2}-\frac{\sqrt{3}}{2}i}{1} = -\frac{1}{2} - \frac{\sqrt{3}}{2} i$$

$x^2 - \binom{\text{Sum of}}{\text{Roots}} x + \binom{\text{Product}}{\text{of Roots}} = 0$

Sum of Roots $= \left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right) + \left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right) = -1 + 0i$

already calculated in division part →

Product of Roots $= \left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)\left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right) = 1 + 0i$

usually use z instead of x in complex equation (either is right)

equation: $z^2 - (-1)x + 1 = 0$

$z^2 + x + 1 = 0$