S.7 P.1 Q.2 a

Show that the sequence with $S_n = pn^2 + qn$ is an arithmetic sequence.
Find the 1st term and the common difference in terms of p and q.

$a = T_1 = S_1$

$S_1 = p(1)^2 + q(1)$
$S_1 = p + q \Rightarrow a = p + q$

$S_2 = p(2)^2 + q(2)$
$= 4p + 2q$

$T_2 = S_2 - S_1$

$S_2 - S_1 = 4p + 2q - p - q$
$\Rightarrow T_2 = 3p + q$

$d = T_2 - T_1$

$d = 3p + \cancel{q} - p - \cancel{q}$
$d = 2p$

If arithmetic
$S_n = \frac{n}{2}[2a + (n-1)d]$
$n = n$
$d = 2p$
$a = p + q$

$S_n = \frac{n}{\cancel{2}}[\cancel{2}(p+q) + (n-1)\cancel{2}p]$

$= n[\cancel{p} + q + pn - \cancel{p}] = pn^2 + qn$ ✓
$\Rightarrow$ its arithmetic.

b (i) Prove if 3 numbers are consecutive terms of a geometric sequence then their logs are consecutive terms of an arithmetic sequence.
Find S_n for the arithmetic sequence if, a is the first term and r is the common ratio of the geometric sequence.

Given: Geometric Sequence: a, ar, ar^2

To prove: $\log a, \log ar, \log ar^2$ is arithmetic ?

$\log ab = \log a + \log b$
$\log a^2 = 2\log a$

$= \log a, \log a + \log r, \log a + \log r^2$
$= \log a, \log a + \log r, \log a + 2\log r$

$d = T_2 - T_1$
$d = T_3 - T_2$

$T_2 - T_1 = \cancel{\log a} + \log r - \cancel{\log a} = \log r$
$T_3 - T_2 = \cancel{\log a} + 2\log r - \cancel{\log a} - \log r = \log r$
$\Rightarrow d = \log r$ sequence is arithmetic

$S_n = \frac{n}{2}[2a + (n-1)d]$
$n = n$
$a = \log a$
$d = \log r$

$S_n = \frac{n}{2}[2\log a + (n-1)\log r]$

b (ii) For a log log to the base of 3 find r if $a = 1$ and $S_{50} = 2450$

Base = 3

$a = 1$

$r = ?$

$n = 50$

$S_n = 2450$

$$S_n = \frac{n}{2}\left[2\log a + (n-1)\log r\right]$$

$$\Rightarrow 2450 = \frac{50}{2}\left[2\log_3 1 + (50-1)\log_3 r\right]$$

$$2450 = 25\left[0 + 49\log_3 r\right]$$

$$2450 = 1225\log_3 r$$

$$2 = \log_3 r$$

$$\Rightarrow r = 3^2 = 9$$

if $\log_b a = n$

$\Rightarrow b^n = a$