

Answer all four questions from this section.

Question 1

(25 marks)

(a) If $z = -3 - i$, where $i = \sqrt{-1}$, find

(i) \bar{z} ,

$$\bar{z} = -3 + i$$

(ii) the quadratic equation with roots z and \bar{z} .

$$z^2 - (\text{Sum roots})z + (\text{product roots}) = 0$$

$$\text{Sum roots} = (-3 - i) + (-3 + i) = -6$$

$$\text{product roots} = (-3 - i)(-3 + i) = 9 - i^2 = 10$$

$$\Rightarrow \text{quadratic is: } z^2 + 6z + 10 = 0$$

(b) z is a root of the cubic equation $az^3 + 22z^2 + bz + 40 = 0$ where $a, b \in \mathbb{R}$.

Show that $a = 3$ and find b .

Write down all of the roots of the cubic equation.

Divide by quadratic in part (ii)

$$\begin{array}{r}
 az^3 + 22z^2 + bz + 40 \\
 \underline{-(az^3 + 6az^2 + 10az)} \\
 (22-6a)z^2 + (b-10a)z + 40 \\
 \underline{-(22-6a)z^2 + (132-36a)z + (22-6a)} \\
 0 \quad + \quad 0
 \end{array}$$

$$\begin{array}{l|l}
 \Rightarrow b - 10a = 132 - 36a & \Rightarrow 40 = 22 - 6a \\
 26a + b = 132 & 60a = 180 \\
 & a = 3 \quad \checkmark
 \end{array}$$

$$26(3) + b = 132 \Rightarrow b = 54$$

$$\text{Other factor} = (az + 22 - 6a) = 3z + 22 - 6(3) = (3z + 4)$$

$$\Rightarrow 3z + 4 = 0 \Rightarrow z = -4/3 \text{ is 3rd Root.}$$

$$\text{Roots: } -3 + i, -3 - i, -4/3$$

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