Question 3 **(25 marks)**

(a) The cubic function $f : x \mapsto x^3 + 7x^2 + 17x + 15$ has one integer root and two complex roots. Find all three roots.

Integer root must be one of: ±1, ±3, ±5. Trial and error yields:

$$f(-3) = (-3)^3 + 7(-3)^2 + 17(-3) + 15 = 0$$

$\therefore$ -3 is a root of *f*, and therefore $x + 3$ is a factor of $f(x)$

$$\begin{array}{r|l} & x^2 + 4x + 5 \\ \hline x+3 & x^3 + 7x^2 + 17x + 15 \\ & \underline{x^3 + 3x^2} \\ & 4x^2 + 17x + 15 \\ & \underline{4x^2 + 12x} \\ & 5x + 15 \\ & \underline{5x + 15} \end{array}$$

Solve $x^2 + 4x + 5 = 0$ to find the two complex roots.

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)} = \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2i}{2} = -2 \pm i$$

Or

$$x^2 + 4x + 5 = 0$$
$$(x+2)^2 = -1$$
$$x + 2 = \pm i$$
$$x = -2 \pm i$$

Roots of *f* are: $-3, \quad -2 + i, \quad -2 - i$

(b) Using part **(a)**, or otherwise, solve the equation $(x-2)^3 + 7(x-2)^2 + 17(x-2) + 15 = 0$.

$x - 2 = -3, \qquad x - 2 = -2 + i, \qquad x - 2 = -2 - i$

$x = -1, \qquad x = i, \qquad x = -i$