Revision Exercise (Core)

1. Find the first four terms of these sequences given the nth term in each case:

(i) $T_n = 3n + 4$
(ii) $T_n = 6n - 1$
(iii) $T_n = 2^{n-1}$
(iv) $T_n = (n + 3)(n + 4)$
(v) $T_n = n^3 + 1$

(i)
$T_1 = 3(1) + 4 = 7$
$T_2 = 3(2) + 4 = 10$
$T_3 = 3(3) + 4 = 13$
$T_4 = 3(4) + 4 = 16$ ✓

(ii)
$T_1 = 6(1) - 1 = 5$
$T_2 = 6(2) - 1 = 11$
$T_3 = 6(3) - 1 = 17$
$T_4 = 6(4) - 1 = 23$ ✓

(iii)
$T_1 = 2^{1-1} = 2^0 = 1$
$T_2 = 2^{2-1} = 2^1 = 2$
$T_3 = 2^{3-1} = 2^2 = 4$
$T_4 = 2^{4-1} = 2^3 = 8$ ✓

(iv)
$T_1 = (1+3)(1+4) = 20$
$T_2 = (2+3)(2+4) = 30$
$T_3 = (3+3)(3+4) = 42$
$T_4 = (4+3)(4+4) = 56$ ✓

(v)
$T_1 = 1^3 + 1 = 2$
$T_2 = 2^3 + 1 = 9$
$T_3 = 3^3 + 1 = 28$
$T_4 = 4^3 + 1 = 65$ ✓

2. The third term of an arithmetic sequence is 71 and the seventh term is 55. Find the first term and the common difference.

$T_n = a + (n-1)d$

$T_3 = 71$
$\Rightarrow a + (3-1)d = 71$
$a + 2d = 71$ ①

$T_7 = 55$
$\Rightarrow a + (7-1)d = 55$
$a + 6d = 55$ ②

Solve ① - ②

$\cancel{a} + 2d = 71$
$-\cancel{a} - 6d = -55$
$-4d = 16 \Rightarrow d = -4$

$\rightarrow$ ①

$a + 2(-4) = 71$
$a - 8 = 71$
$a = 79$ ✓

3. In a geometric series, the first term is 12 and the sum to infinity is 36. Find the common ratio.

$a = 12 \quad S_\infty = 36 \quad r = ?$

$S_\infty = \frac{a}{1-r}$

$\Rightarrow 36 = \frac{12}{1-r}$

multiply by (1 - r)
divide by 36

$\Rightarrow 1 - r = \frac{12}{36}$

$1 - r = \frac{1}{3}$

subtract 1

$-r = \frac{1}{3} - 1 = -\frac{2}{3}$

Change signs

$r = \frac{2}{3}$ ✓

4. Find the common ratio in each of the following geometric progressions and hence write an expression for T_n, the nth term.

(i) $-2, 4, -8, \ldots$
(ii) $1, \frac{1}{2}, \frac{1}{4}, \ldots$
(iii) $2, -6, 18, \ldots$

$r = \frac{T_2}{T_1}$

$T_n = ar^{n-1}$

(i) $r = \frac{4}{-2} = -2$, $a = -2$

$\Rightarrow T_n = (-2)(-2)^{n-1} = (-2)^n$

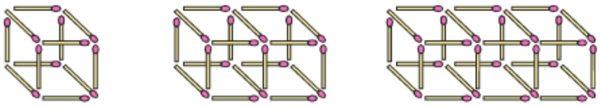
(ii) $r = \frac{(\frac{1}{2})}{1} = \frac{1}{2}$, $a = 1$

$\Rightarrow T_n = (\frac{1}{2})^{n-1}$

(iii) $r = -6/2 = -3$, $a = 2$

$\Rightarrow T_n = 2(-3)^{n-1}$ ✓

5. Using matchsticks, a series of cubes are made and joined as cuboids, as shown in the diagram.



(i) Determine the number of matchsticks needed for the nth cuboid.
(ii) Determine the maximum number of cubes in the cuboid if there are 2006 matchsticks left for the construction.

(i) $T_1 = 12$ matches $\Rightarrow a = 12$

$T_2 = 12+8 = 20$ matches

$T_3 = 20+8 = 28$ matches $d = 8$

$T_n = a+(n-1)d$

$T_n = 12+(n-1)8$

$= 12+8n-8$

$T_n = 8n+4$

$T_n = 2006$
$n = ?$

(ii) $8n+4 = 2006$

$8n = 2002$

$n = 2002/8 = 250\frac{1}{4}$

$\Rightarrow$ max. cuboid $= 250$ ✓

6. The second term of a geometric sequence is 21.
The third term is -63.
Find (i) the common ratio (ii) the first term.

$T_n = ar^{n-1}$

$T_2 = 21$ $T_3 = -63$

$r = \frac{T_3}{T_2}$

$\Rightarrow r = \frac{-63}{21}$ $\Rightarrow r = -3$

Sequence: $a \xrightarrow{\times -3} 21 \xrightarrow{} -63$

$\Rightarrow a = \frac{21}{-3}$ $\Rightarrow a = -7$ ✓

7. €2000 is invested in a savings scheme which offers 2.5% compound interest. Explain how the expression $A = €2000(1.025)^5$ represents the value of the investment after 5 years.

A = amount r = rate

P = principle I = interest

t = time $P(r) = I$

Year 1 $A_1 = P + I = P + P(r) = P(1+r)$

$A_1 = P_2$ Year 2 $A_2 = P_2 + I = P(1+r) + P(1+r)r$

$= P(1+r)(1+r)$

$= P(1+r)^2$ etc..

This is a geometric sequence:

$A = P(1+r)^t$

8. Find the sum of the first 200 natural numbers.

arithmetic series

$S_n = \frac{n}{2}[2a + (n-1)d]$

$a = 1$
$d = 1$
$n = 200$

$S_{200} = \frac{200}{2}[2(1) + (200-1)1]$

$= 100[2 + 199]$

$= 100[201]$

$= 20,100$ ✓

9. The fifth term of an arithmetic sequence is twice the second term.
The two terms also differ by 9.
Find the sum of the first 10 terms of the sequence.

let $T_2 = x$
$T_5 = 2x$

$T_5 - T_2 = 9 \Rightarrow 2x - x = 9$
$x = 9$

$\Rightarrow T_2 = 9 \quad , \quad T_5 = 18$

$T_n = a + (n-1)d$

$a + 1d = 9 \quad (1) \quad , \quad a + 4d = 18 \quad (2)$

$(2) - (1)$

$3d = 9 \Rightarrow d = 3$

$\to (1)$

$a + 3 = 9 \Rightarrow a = 6$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}\left[2(6) + (10-1)3\right] = 195$

10. Evaluate $\sum_{r=3}^{16} \boxed{(2r + 1).}$ ← Term formula

note for $T_1 \Rightarrow r = 3$

$T_1 = 2(3)+1 = 7$
$T_2 = 2(4)+1 = 9$
$T_3 = 2(5)+1 = 11$

$a = 7$
$d = 2$

How many Terms?

From term $r=3$ to $r=16$ there are 14 terms
$\Rightarrow n = 14$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{14} = \frac{14}{2}\left[2(7) + (14-1)2\right] = 7\left[14 + 26\right] =$
$= 7[40] = 280$