j

Sequences –Series – Patterns

chapter 4

Key words

number sequence arithmetic sequence series sigma (Σ)
geometric sequence exponential sequence geometric series recurring decimal
finite difference composite function quadratic function

6th Year

HL Maths

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Example 1

Write down the first four terms of each of the following sequences:

(i) $T_n = n^2 + n$ (ii) $T_n = 2^n - 3n$

(i) $T_n = n^2 + n$

$T_1 = (1)^2 + (1) = 2$

$T_2 = (2)^2 + 2 = 6$

$T_3 = (3)^2 + 3 = 12$

$T_4 = (4)^2 + 4 = 20$

(ii) $T_n = 2^n - 3n$

$T_1 = 2^1 - 3(1) = -1$

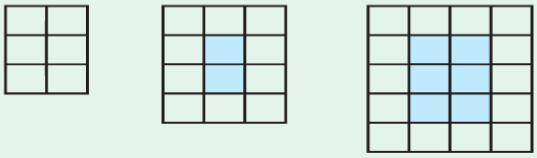
$T_2 = 2^2 - 3(2) = -2$

$T_3 = 2^3 - 3(3) = -1$

$T_4 = 2^4 - 3(4) = 4$

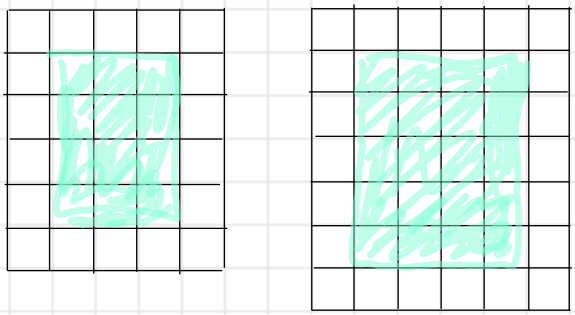
Example 2

The following rectangular patterns are made from two sets of coloured tiles.



(i) Draw the next two patterns of tiles.
(ii) Write a number sequence for the blue tiles used in each of these patterns.
(iii) Write a number sequence for the total number of tiles used in each of these patterns.
(iv) Write a number sequence for the white tiles used in each of these patterns.
(v) Write out the next 3 terms in each sequence found in (ii), (iii), (iv).

(i)



(ii) Blue: 0, 2, 6, 12, 20, 30, 42, 56

(v) (iii) All tiles: 6, 12, 20, 30, 42, 56, 72, 90

(iv) White tiles: 6, 10, 14, 18, 22, 26, 30, 34

Exercise 4.1

1. Write down the next three terms of each of the following sequences:

arithmetic +6 (i) 6, 12, 18, 24, ... 30, 36, 42

(ii) 7, 12, 17, 22, ...

(iii) 4.7, 5.9, 7.1, 8.3, ...

arithmetic −3 (iv) 2, −1, −4, −7, ... −10, −13, −16

(v) 2, 3, 6, 11, 18, 27, ...

arithmetic −8 (vi) 78, 70, 62, 54, ... 46, 38, 30

(vii) 10, 5, 0, −5, −10, ...

(viii) −64, −55, −46, −37, ...

(ix) 2, 6, 18, ...

quadratic +4, +6, +8 ... (x) 2, 6, 12, 20, ... 30, 42, 56

arithmetic $-\frac{1}{2}$ (xi) $\frac{3}{4}, \frac{1}{4}, -\frac{1}{4}, -\frac{3}{4}, -\frac{5}{4}, -\frac{7}{4}$

(xii) 1, 2, 4, 7, 11, ...

quadratic +3, +5, +7 ... (xiii) 0, 3, 8, 15, 24, ... 35, 48, 63

geometric ×−2 (xiv) 3, −6, 12, −24, ... 48, −96, 192

(xv) $\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \ldots$

2. Find the first four terms of the following sequences, given the nth term (T_n) in each case.

(i) $T_n = 4n - 2$ (iv) $T_n = (n + 3)(n + 1)$ (vii) $T_n = 2^n$

(ii) $T_n = (n + 1)^2$ (v) $T_n = n^3 - 1$ (viii) $T_n = (-3)^n$

(iii) $T_n = n^2 - 2n$ (vi) $T_n = \frac{n}{n+2}$ (ix) $T_n = n.2^n$

HW

(i) $T_n = 4n-2$

$T_1 = 4(1)-2 = 2$

$T_2 = 4(2)-2 = 6$

$T_3 = 4(3)-2 = 10$

$T_4 = 4(4)-2 = 14$ ✓

(ii) $T_n = (n+1)^2$

$T_1 = (1+1)^2 = 4$

$T_2 = (2+1)^2 = 9$

$T_3 = (3+1)^2 = 16$

$T_4 = (4+1)^2 = 25$ ✓

(iii) $T_n = n^2 - 2n$

$T_1 = (1)^2 - 2(1) = -1$

$T_2 = (2)^2 - 2(2) = 0$

$T_3 = (3)^2 - 2(3) = 3$

$T_4 = (4)^2 - 2(4) = 8$ ✓

(iv) $T_n = (n+3)(n+1)$

$T_1 = (1+3)(1+1) = 8$

$T_2 = (2+3)(2+1) = 15$

$T_3 = (3+3)(3+1) = 24$

$T_4 = (4+3)(4+1) = 35$ ✓

(v) $T_n = n^3 - 1$

$T_1 = (1)^3 - 1 = 0$

$T_2 = (2)^3 - 1 = 7$

$T_3 = (3)^3 - 1 = 26$

$T_4 = (4)^3 - 1 = 63$ ✓

(vi) $T_n = n/(n+2)$

$T_1 = 1/(1+2) = 1/3$

$T_2 = 2/(2+2) = 1/2$

$T_3 = 3/(3+2) = 3/5$

$T_4 = 4/(4+2) = 2/3$ ✓

2. Find the first four terms of the following sequences, given the nth term (T_n) in each case.

(i) $T_n = 4n - 2$ (iv) $T_n = (n + 3)(n + 1)$ (vii) $T_n = 2^n$

(ii) $T_n = (n + 1)^2$ (v) $T_n = n^3 - 1$ (viii) $T_n = (-3)^n$

(iii) $T_n = n^2 - 2n$ (vi) $T_n = \frac{n}{n+2}$ (ix) $T_n = n.2^n$

HW

(vii) $T_n = 2^n$

$T_1 = 2^1 = 2$

$T_2 = 2^2 = 4$

$T_3 = 2^3 = 8$

$T_4 = 2^4 = 16$ ✓

(viii) $T_n = (-3)^n$

$T_1 = (-3)^1 = -3$

$T_2 = (-3)^2 = 9$

$T_3 = (-3)^3 = -27$

$T_4 = (-3)^4 = 81$ ✓

(ix) $T_n = n.2^n$

$T_1 = (1)2^1 = 2$

$T_2 = (2)2^2 = 8$

$T_3 = (3)2^3 = 24$

$T_4 = (4)2^4 = 64$ ✓

Arithmetic Sequence

eg. T_1, T_2, T_3, T_4, T_5

$2, 4, 6, 8, 10 \ldots$

+2 +2 +2 +2 +2

T_n = n^{th} term eg.. $T_3 = 6$

n = n

a = T_1 = 2

d = common difference = +2

$T_{20} = ?$ $2 + 19(2)$

$T_{99} = ?$ $2 + 98(2)$

$T_n = ?$ $T_n = 2 + (n-1)2$

Formula $T_n = a + (n-1)d$

Example 1

Find the nth term (T_n) of the arithmetic sequence:

$-2, \quad 3, \quad 8, \quad 13, \ldots\ldots$

and hence find (i) T_{20} (ii) T_{21} (iii) $T_{21} - T_{20}$.

$T_n = a + (n-1)d$

$a = -2$ $d = 5$.

$n = n$

$T_n = -2 + (n-1)5$

$= -2 + 5n - 5$

$T_n = -7 + 5n$

$n = 20$ (i) $T_{20} = -7 + 5(20) = 93$

(ii) $T_{21} = -7 + 5(21) = 98$

(iii) $T_{21} - T_{20} = 5$

Exercise 4.2

1. Find T_n, the nth term of the following arithmetic sequences. Hence find T_{22} for each sequence.

(i) 8, 13, 18, 23, ... (ii) 16, 36, 56, 76, ... (iii) 10, 7, 4, 1, ...

(i) $a = 8 \quad d = 5$

$T_n = 8 + (n-1)5$

$T_n = a + (n-1)d$ $\quad T_n = 8 + 5n - 5$

$T_n = 3 + 5n$

$n = 22$ $\quad T_{22} = 3 + 5(22) = 113$

(ii) $a = 16 \quad d = 20$

$T_n = 16 + (n-1)20$

$= 16 + 20n - 20$

$T_n = 20n - 4$

$n = 22$ $\quad T_{22} = 20(22) - 4 = 436$

Exercise 4.2

1. Find T_n, the nth term of the following arithmetic sequences. Hence find T_{22} for each sequence.

(i) 8, 13, 18, 23, ... (ii) 16, 36, 56, 76, ... (iii) 10, 7, 4, 1, ...

(iii) 10, 7, 4, 1 ...

$a = 10 \quad d = -3$

$T_n = a + (n-1)d$ $\quad T_n = 10 + (n-1)(-3) = 10 - 3n + 3$

$T_n = 13 - 3n$

$n = 22$ $\quad T_{22} = 13 - 3(22) = -53$ ✓

3. Find the number of terms in each of the following arithmetic sequences:
(i) $-5, -1, 3, 7, \ldots\ldots 75$ (ii) $2, 5, 8, 11, \ldots\ldots 59$ (iii) $-\frac{3}{2}, -1, -\frac{1}{2}, 0, \ldots\ldots 14.$

(i)

$T_n = a + (n-1)d$

$n = ?$

$a = -5 \quad d = 4 \quad T_n = 75 \quad n = ?$

$\Rightarrow 75 = -5 + (n-1)4$

$75 = -5 + 4n - 4$
$75 = -9 + 4n$
$84 = 4n$
$21 = n$ ✓

$\Rightarrow T_{21} = 75$

check: $T_{21} = -5 + (20)4 = 75$ ✓

3. Find the number of terms in each of the following arithmetic sequences:
(i) $-5, -1, 3, 7, \ldots\ldots 75$ (ii) $2, 5, 8, 11, \ldots\ldots 59$ (iii) $-\frac{3}{2}, -1, -\frac{1}{2}, 0, \ldots\ldots 14.$

(ii)

$n = ?$

$T_n = a + (n-1)d$

$a = 2, \quad d = 3, \quad T_n = 59, \quad n = ?$

$59 = 2 + (n-1)3$
-2
$57 = (n-1)3$
expand
$57 = 3n - 3$
$+3$
$60 = 3n$
$\div 3$
$20 = n$

$\Rightarrow T_{20} = 59$ ✓

check: $T_{20} = 2 + (19)3 = 59$ ✓

3. Find the number of terms in each of the following arithmetic sequences:

(i) $-5, -1, 3, 7, \ldots\ldots 75$ (ii) $2, 5, 8, 11, \ldots\ldots 59$ (iii) $-\frac{3}{2}, -1, -\frac{1}{2}, 0, \ldots\ldots 14.$

(ii)

$n = ?$

$T_n = a + (n-1)d$

$a = -\frac{3}{2}$, $d = \frac{1}{2}$, $T_n = 14$, $n = ?$

$\Rightarrow \quad 14 = -\frac{3}{2} + (n-1)\left(\frac{1}{2}\right)$

$\times 2 \quad 28 = -3 + (n-1)1$

$+3 \quad 31 = n - 1$

$+1 \quad 32 = n$

$\Rightarrow \quad T_{32} = 14$ ✓

check: $T_{32} = -\frac{3}{2} + (31)\left(\frac{1}{2}\right) = 14$ ✓