Example 2

To celebrate the birth of his niece, an uncle offers to open a savings account with a deposit of €50. He also offers to every year add €10 more than he did the previous year until his niece is 21 years of age.

(i) Find an expression for S_n, the sum of money on deposit after n years.

(ii) Find S_{22}, the total saved after 21 years.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$50 + 60 + 70 + \ldots + \ldots$

$T_1 = a = 50$, $n = 22$, $d = 10$

$S_n = \frac{n}{2}[2(50) + (n-1)10]$

$= \frac{n}{2}[100 + 10n - 10]$

$= \frac{n}{2}[90 + 10n] = n[45 + 5n]$

$= 5n^2 + 45n$

$S_{22} = 5(22)^2 + 45(22) = €3,410$

5. Anna saves money each week to buy a printer which costs €190. Her plan is to start with €10 and to put aside €2 more each week (i.e. €12, €14, etc.) until she has enough money to buy the printer.
At this rate, how many weeks will it take Anna to save for the printer?

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = 190$
$a = 10$
$d = 2$
$n = ?$

$190 = \frac{n}{\cancel{2}}[\cancel{2}(10) + (n-1)\cancel{2}]$

$190 = n[9 + n] = 9n + n^2$

Solve

$n^2 + 9n - 190 = 0$
$(n + 19)(n - 10) = 0$
$n = -19, 10$

Ans: 10 weeks

Example 3

Given $S_n = n^2 - 4n$, find an expression for T_n and hence determine if the sequence is arithmetic.

$a = T_1 = S_1 = (1)^2 - 4(1) = -3$

$S_2 = T_1 + T_2 = T_2 - 3 = (2)^2 - 4(2) = -4$

$T_2 - 3 = -4 \Rightarrow T_2 = -1$

$d = ?$ $\quad T_2 - T_1 = -1 - -3 = +2$

$T_n = a + (n-1)d$

$T_n = -3 + (n-1)2$

$= -3 + 2n - 2$

$T_n = 2n - 5$

Arithmetic?

$T_{n+1} - T_n = k$

$T_{n+1} = 2(n+1) - 5 = 2n + 2 - 5$

$T_{n+1} = 2n - 3$

$T_{n+1} - T_n = 2n - 3 - (2n - 5)$

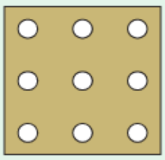
$= 2$ $\quad$ = const ⇒ ARITHMETIC

Example 4

A lighting company is making a sequence of light panels with the number of bulbs per panel in arithmetic sequence.

For the first 10 panels, 165 bulbs were used.

If the third panel is as shown in the diagram, find a, the first term of the sequence, and d, the common difference.



3rd panel (9 bulbs)

Hence draw a diagram of the first four panels.

$S_{10} = 165$ $\quad n = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$165 = 5[2a + 9d]$

$33 = 2a + 9d$

$T_n = a + (n-1)d$

$T_3 = 9$ $\quad 9 = a + 2d$

$n = 3$

Example 5

(i) Use the sigma notation ($\sum$) to represent $2 + 6 + 10 + 14 + \ldots$ for 45 terms.

Seris

(ii) For what value of n is $\sum_{r=1}^{n} 3r - 5 = 90$?

(iii) Find the value of $\sum_{r=1}^{8} 4r - 1$.

Σ = sigma = "Sum"

end → n

$\sum_{n=1}^{n} T_n$ ← Term Rule

start

(i) $a=2$, $d +4$, $n=n$, T_n

$T_n = a+(n-1)d$

$T_n = 2+(n-1)4$

$T_n = 2+4n-4 = 4n-2$

$\sum_{n=1}^{45} 4n-2$

Example 5

(i) Use the sigma notation ($\sum$) to represent $2 + 6 + 10 + 14 + \ldots$ for 45 terms.

(ii) For what value of n is $\sum_{r=1}^{n} (3r - 5) = 90$?

(iii) Find the value of $\sum_{r=1}^{8} 4r - 1$.

$\boxed{S_n = 90}$

$T_n = 3n-5$

$a=-2$

$a = T_1 = 3(1)-5 = -2$

$T_2 = 3(2)-5 = 1$ $+3$

$d=3$

$n=?$

$S_n = \frac{n}{2}[a+(n-1)d]$

$90 = \frac{n}{2}[-2+(n-1)3]$

Example 5

(i) Use the sigma notation ($\sum$) to represent $2 + 6 + 10 + 14 + \ldots..$ for 45 terms.

(ii) For what value of n is $\sum_{r=1}^{n} 3r - 5 = 90$?

(iii) Find the value of $\sum_{r=1}^{8} 4r - 1$.

$S_8 = ?$

$a = T_1 = 4(1) - 1 = 3$ $\Big) +4$

$T_2 = 4(2) - 1 = 7$

$d = 4$

$n = 8$

6. Evaluate (i) $\sum_{r=1}^{6} (3r + 1)$ (ii) $\sum_{r=0}^{5} (4r - 1)$ (iii) $\sum_{r=1}^{100} r$

$S_6 = ?$

$a = T_1 = 3(1) + 1 = 4$ $\Big) +3$

$T_2 = 3(2) + 1 = 7$

$d = 3$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_6 = \frac{6}{2}\left[2(4) + 5(3)\right]$

$= 3[8 + 15]$

$= 69$