Example 1

LHS RHS

Prove that **for all values of *n***, $1 + 2 + 3 + 4 + \dots n = \frac{n}{2}(n+1)$.

① $n=1$

LHS $\stackrel{?}{=}$ RHS

$1 = \frac{1}{2}(1+1) = \frac{1}{2}(2) = 1$ ✓ yes

② Assume true for $n=k$

Assume : $1+2+\dots+k = \frac{k}{2}(k+1)$

③ To prove for $n=k+1$

Aim to show Sum to $k+1$ $= \frac{(k+1)}{2}((k+1)+1)$

Add $k+1$ to both sides

$\Rightarrow$ RHS $= \frac{k}{2}(k+1) + (k+1)$

$= \frac{k^2}{2} + \frac{k}{2} + k + 1$

$= \frac{1}{2}[k^2 + k + 2k + 2]$

$= \frac{1}{2}(k^2 + 3k + 2) = \frac{1}{2}(k+2)(k+1)$

$= \frac{k+1}{2}(k+2) = \frac{k+1}{2}(k+1+1)$

④ Conclude true for all values of n

Its true $n=k$, $n=k+1$, and $n=1$

$\Rightarrow$ its true for $n = 2, 3, 4 \dots$ and all values

Exercise 7.12(A)

In each of the following questions, prove the results by mathematical induction for all positive integer values of n.

1. $2 + 4 + 6 + 8 + \dots 2n = \sum_{n=1}^{n} 2n = n(n+1)$.

$n=1$?

LHS $\stackrel{?}{=}$ RHS

$2 = 1(1+1)$

$2 = 2$ ✓ true

Assume : $\sum_{k=1}^{k} 2k = k(k+1)$

$n=k+1$?

Is Sum $= (k+1)(k+1+1)$

$= (k+1)(k+2)$?

term $k+1$ $= 2(k+1)$

Sum to $n=k+1$ $= k(k+1) + \text{term } k+1$

$= k(k+1) + 2(k+1)$

$= (k+1)(k+2)$ ✓ true

It is true for n = 1, n = k and n = k + 1

so it is also true for n = 2, 3, 4... and all natural numbers

LHS RHS

2. $1 + 4 + 7 + 10 + \ldots. (3n - 2) = \frac{n}{2}(3n - 1)$.

① $n=1$

LHS $\overset{?}{=}$ RHS

$1 = $

② Assume true for $n=k$

Assume: $1+4+\ldots(3k-2) = \frac{k}{2}(3k-1)$

③ To prove for $n=k+1$

Aim to show sum to $k+1$ $= \frac{(k+1)}{2}(3(k+1)-1)$

Add $k+1$ term to both sides ie.. $3(k+1)-2$
$= 3k+3-2 = 3k+1$

RHS $= \frac{k}{2}(3k-1) + 3k+1$

$= \frac{3k^2}{2} - \frac{k}{2} + 3k+1$

$= \frac{1}{2}[3k^2+5k+2]$

$= \frac{1}{2}(3k+2)(k+1) = \frac{(k+1)}{2}(3(k+1)-1)$

④ Conclude true for all values of n

Its true $n=k$, $n=k+1$, and $n=1$
$\Rightarrow$ its true for $n=2,3,4\ldots$ and all values

3. $1.2 + 2.3 + 3.4 + 4.5 + \ldots. n(n+1) = \sum_{n=1}^{n} n(n+1) = \frac{n}{3}(n+1)(n+2)$.

① $n=1$?

$n=1$ LHS $\overset{?}{=}$ RHS

$1(2) = \frac{1}{3}(1+1)(1+2)$

$2 = \cancel{\frac{1}{3}}(2)(\cancel{3})$ true.

② Assume for $n=k$

Assume sum to term $n=k$ $= \frac{k}{3}(k+1)(k+2)$

③ Is sum to $n=k+1$ $= \frac{(k+1)}{3}(k+1+1)(k+1+2)$

term $(k+1)$
$= (k+1)(k+1+1)$
$= (k+1)(k+2)$

Sum to $n=k$ $= \frac{k}{3}(k+1)(k+2) + \text{term}(k+1)$

$= \frac{k}{3}(k+1)(k+2) + (k+1)(k+2)$

$= \left(\frac{k}{3}+1\right)(k+1)(k+2)$

$= \frac{(k+3)(k+1)(k+2)}{3}$

$= \frac{(k+1)}{3}((k+1)+1)((k+1)+2)$

$\Rightarrow$ its true for $n=k+1$

④ State true for all $n \in \mathbb{N}$

Since it is true for $n=k$, $n=k+1$, $n=1$
$\Rightarrow$ true for $n = 2,3,4\ldots$
$\Rightarrow$ true for all $n \in \mathbb{N}$