Co-Ordinate Geometry – The Line

Points
If we have **two points** (x_1, y_1) and (x_2, y_2) we can use formulae in the tables to find:

- **Distance** $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- **Midpoint** $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
- **Slope** $m = \frac{y_2-y_1}{x_2-x_1}$
- **Equation of a line** $y - y_1 = m(x - x_1)$

Perpendicular Distance from a Point to a Line

$$d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

Slopes

Perpendicular Slope
Turn the slope upside down and change the sign.
For example if a line has a slope of $\frac{3}{5}$ the perpendicular slope is $-\frac{5}{3}$
To prove slopes perpendicular $m_1 m_2 = -1$

Parallel Slope
If the lines are parallel then the slopes are equal

Angles Between Lines

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$$

(remember: use shift on the calculator to find an angle)

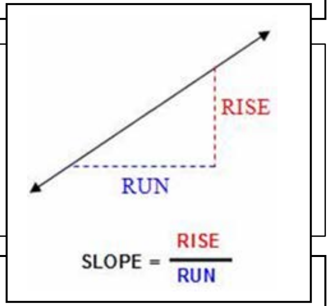
Line in the form $ax + by + c = 0$

We can find:

- where it **crosses the x axis** by letting $y = 0$ (also do this to **draw a line**)
- where it **crosses the y axis** by letting $x = 0$
- the **slope m,** using $-\frac{a}{b}$
- if a **point is on the line** by subbing the values of the point (x_1, y_1) in for x and y.

Line in the form $y = mx + c$

m will be the slope (rise over run)
c the y-intercept (the place where the line crosses y axis)



Area of a triangle
If given the 3 vertices of the triangle we can use the formula in the tables. We move one of the points to (0,0) and the others the same distance through translation.

$(2,4) \to (0,0)$
$(3,-3) \to (1,-7)$ Area of a triangle $\frac{1}{2}|x_1y_2 - x_2y_1|$
$(-3,1) \to (-5,-3)$

Or you can use $\frac{1}{2}$ **base** $\times$ **height** if we somehow find the base and height.

The modulus symbol $|\square|$ means that we take the positive value of the answer.

Dividing Line in the Ratio $m:n$

$$p = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$
Internally

$$p = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$
Externally

Remember
For both the line and circle questions:

- Open the relevant **formula pages in the tables.**
- Always **draw a rough sketch**. Remember to sketch a line you must find two points and the easiest to find are where $x = 0$ and $y = 0$
- If you need a bit from one question to do the next but don't have it, make your best guess and use them. Always show the examiner what you CAN do.

Transformations of the Plane

1. Translation: A translation moves a point in straight line.
2. Central Symmetry: Is a reflection in a point
3. Axial Symmetry: Is a reflection in a line
4. Axial Symmetry in the axes or central symmetry in the origin
   a. In the $x - axis$ – change the sign of y
   b. In the $y - \ axis$ – change the sign of x
   c. Central symmetry

Graphing Lines
To draw a line two points are needed. The easiest points to find are where the lines cross the x and y axis.

1. Let $y = 0$ and find x
2. Let $x = 0$ and find y
3. Plot these two points
4. Draw the line through these points

Point of Intersection of Two Lines
Find the point of intersection of the two lines
K: $3x + 4y = -6$ and L: $2x - 3y = 13$

Simultaneous Equation

Lines Parallel to the Axes
$x = 2$ is a line parallel to the $y - axis$ through 2 on the x axis
$y = -1$ is a line parallel to the $x - axis$ through -1 on the y axis

Parallel Lines
If $ax + by + c = 0$ is a line then a **parallel** line can be written $ax + by + k = 0$
A perpendicular line can be written $bx - ay + k = 0$

Coordinates of a Centroid
$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Circumcentre
Point of intersection of the perpendicular bisectors of the sides.

Orthocentre
Is the point of intersection of the perpendicular lines from the vertices to the opposite sides.

A **translation** moves a point a given distance and direction by:

A given rule
Find the image of the point $(3,1)$ through the translation $(2, -1) \to (4,1)$
$(3,1) \to (5,3)$ under the above translation (x number up 2, y number up 2)

Symmetry
An example would be to find image of the point $(3,1)$ through central symmetry through $(1,2)$
$$(3,1) \to (1,2) \to (-1,3)$$