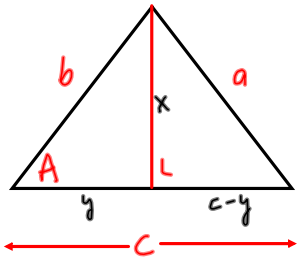
Prove the Cosine Rule, $a^2 = b^2 + c^2 - 2bc\cos A$



[Pythagoras] $b^2 = x^2 + y^2$

[Pythagoras] $a^2 = x^2 + (c-y)^2$

$a^2 = x^2 + c^2 - 2cy + y^2$

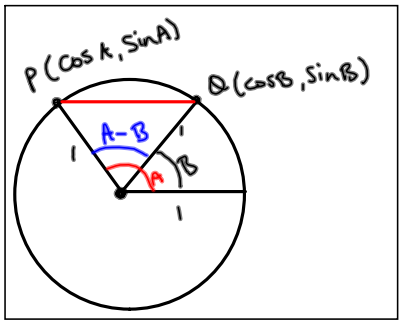
$\Rightarrow a^2 = b^2 + c^2 - 2cy$

$\cos A = \frac{y}{b} \Rightarrow y = b\cos A$

$\Rightarrow a^2 = b^2 + c^2 - 2bc\cos A$

Derive (4.) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

Unit circle



Using distance formula

$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|PQ|^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$= (\cos A - \cos B)^2 + (\sin A - \sin B)^2$

$= \cos^2 A - 2\cos A\cos B + \cos^2 B + \sin^2 A - 2\sin A\sin B + \sin^2 B$

But $\cos^2 A + \sin^2 A = 1$

$\Rightarrow |PQ|^2 = 2 - 2(\cos A\cos B - \sin A\sin B)$

Using Cosine Rule $|PQ|^2 = 1^2 + 1^2 - 2(1)(1)\cos(A-B)$

$|PQ|^2 = 2 - 2(\cos(A-B))$

$\Rightarrow \cos(A-B) = \cos A\cos B - \sin A\sin B$ QED